

# Adaptive Control of Space Based Robot Manipulators

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## Abstract

For space based robots in which the base is free to move, motion planning and control is complicated by uncertainties in the inertial properties of the manipulator and its load. This paper presents a new adaptive control method for space based robots which achieves globally stable trajectory tracking in the presence of uncertainties in the inertial parameters of the system.

The paper begins with a partitioning of the fifteen degree of freedom system dynamics into two components: a nine degree of freedom invertible portion and a six degree of freedom noninvertible portion. The controller is then designed to achieve trajectory tracking of the invertible portion of the system. This portion of the system consist of the manipulator joint positions and the orientation of the base. The motion of the noninvertible portion is bounded, but unpredictable. This portion of the system consist of the position of the robot's base and the position of the reaction wheels.

## 1 Introduction

In recent years the control of space based manipulators has received increased attention. The main difference between space based robots and their terrestrial counterpart is the dynamic coupling between the manipulator and its floating base. This results in a similar, but uniquely different form for the kinematic and dynamic equations of motion.

Several researchers have focused on the forward and inverse kinematics problem [1, 2, 3, 4]. One of the interesting parts of these results is the formulation of the *dynamic* Jacobian matrix. It is now recognized that singularities may occur in the transformation from end-effector velocities to joint velocities which are at different locations than the normal kinematic singularity points. Another interesting result is the concept of the virtual manipulator, [2]. If the mass properties of each link are known, then it can be shown that a virtual manipulator can be obtained for use in the inverse kinematics problem. The advantage of the virtual manipulator is that algorithms developed for terrestrial based manipulators can be applied.

The dynamics of multibody space based systems has been researched for many years [5, 6, 7]. In many ways the control of space based robots is similar to the problems traditionally faced in satellite control. The main difference is the articulated nature

of the robot. Free floating space based robot control has only recently gained attention [8, 9].

In both the kinematics problem and the control problems previous researchers have assumed either the mass properties of the system are completely known or the momentum of the system is zero. This paper presents a control method in which neither of the assumptions are made.

We begin this paper with a description of the system considered and the formulation of the dynamic equations of motion. The fifteen degree of freedom system dynamics are partitioned into two components: a nine degree of freedom invertible portion and a six degree of freedom noninvertible portion. The invertible portion of the system consist of the manipulator joint positions and the orientation of the base. The motion of the noninvertible portion is bounded, but unpredictable. This portion of the system consist of the position of the robot's base and the velocity of the reaction wheels. An adaptive controller is then presented to achieve trajectory tracking of the invertible portion of the system. Finally, a summary of the main results and conclusions of the paper are presented.

## 2 Equations of Motion

The system we are considering is an  $n$  degree of freedom serial link manipulator, with rotational or translational joints, mounted on a base containing three reaction wheels. It is assumed that no external forces or moments are applied to the system. However, no assumptions have been made concerning the initial momentum of the system.

Associated with each link is a right handed Cartesian coordinate system whose position and orientation is fixed with respect to the associated link. This is illustrated for link  $j$  in Figure 1. The location of this coordinate frame with respect to an inertial reference frame is denoted by the homogeneous transform  $T_j$ .

A Floating Referenced Frame is fixed at a specified position on the Base with the same orientation as the inertial reference frame. Its location is denoted by the homogeneous transform  $T_0$ .

The Base link is numbered 3 and its coordinate frame is located at the same location as the Floating Reference Frame, but at a different orientation. Its location is denoted by the

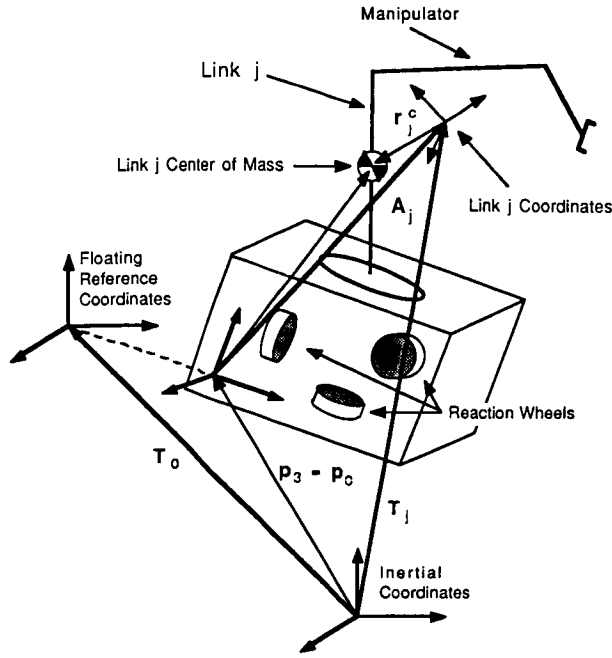


Figure 1: Illustration of Robot

homogeneous transform,  $T_3$ . Between the Floating Reference Frame and the Base frame are two fictitious links of zero mass. The three joints between these links represent the relative change in orientation of the Base with respect to the Floating Reference Frame.

The manipulator is attached to the Base and the links are numbered from 4 to  $n+3$ . In addition three reaction wheels are located inside the Base. The wheels are numbered from  $n+4$  to  $n+6$ .

The configuration of the complete system is a tree structure as illustrated in figure 2 for the case of  $n=6$ . Considering the Floating Reference Frame to be the base of the tree, each joint in the system is numbered the same as the immediate descendant in the tree. The position of the  $i$ -th joint is denoted by  $q_i$ .

The kinetic energy of the system is given by the following equation.

$$K = \sum_{i=3}^{n+6} \frac{1}{2} TR\{\dot{T}_i D_i \dot{T}_i^T\} \quad (1)$$

where  $D_i$  is the constant pseudo inertia matrix for link  $i$  referred to link  $i$  coordinates. [10, 11], and  $TR\{\}$  denotes the trace operator.

$$D_i = \int \mathbf{r} \mathbf{r}^T dm = \begin{bmatrix} \int x^2 dm & \int xy dm & \int xz dm & \int x dm \\ \int xy dm & \int y^2 dm & \int yz dm & \int y dm \\ \int xz dm & \int yz dm & \int z^2 dm & \int z dm \\ \int x dm & \int y dm & \int z dm & \int dm \end{bmatrix}$$

where the integration is carried out over the entire link, and  $\mathbf{r} = [x, y, z, 1]^T$  is the position vector of the mass element with respect to link  $i$  coordinates.

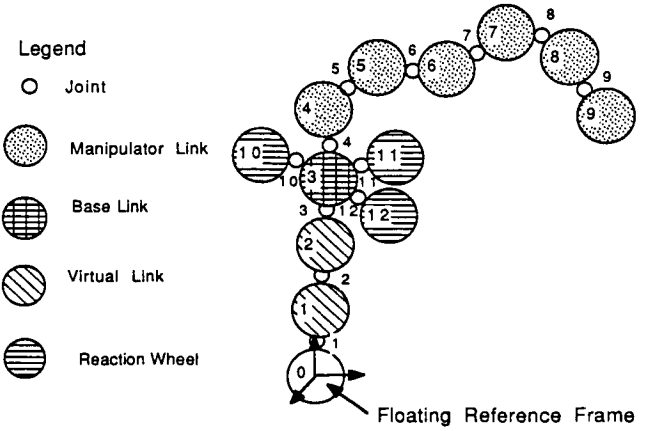


Figure 2: Configuration of Space Based Robot

## 2.1 Reaction Wheels

The torque delivered to the base of the robot to control its orientation is provided by a set of reaction wheels. The position variables associated with these wheels are cyclic and therefore it considerably simplifies the analysis by writing the kinetic energy and the resulting equations of motion in terms of the generalized momentum,  $l_j$ , associated with these wheels.

$$l_j = \frac{\partial K}{\partial \dot{q}_j}$$

The main objective of this section is to write the kinetic energy in terms of these generalized momentum. To this end, we consider the kinetic energy of the  $j$ -th reaction wheel,

$$K_j = \frac{1}{2} TR\{\dot{T}_j D_j \dot{T}_j^T\} \quad (2)$$

We will show this can be written in the following form:

$$K_j = \frac{1}{2} TR\{\dot{T}_3 E_j \dot{T}_3^T\} + \frac{1}{2} J_j^{-1} l_j^2$$

where  $J_j$  is the moment of inertia of the  $j$ -th reaction wheel about its axis of rotation and  $E_j$  is a constant matrix.

We begin with some notation. Let  $\mathbf{x}$  be an arbitrary  $6 \times 1$  vector, which has been partitioned into two  $3 \times 1$  vectors,  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$$

and define the matrix function  $\mathbf{R}(\mathbf{x})$  as

$$\mathbf{R}(\mathbf{x}) = \begin{bmatrix} \mathbf{k}(\mathbf{a}) & \mathbf{b} \\ 000 & 0 \end{bmatrix}$$

where  $\mathbf{k}()$  is a  $3 \times 3$  matrix function such that for any two  $3 \times 1$  vectors  $\mathbf{a}$  and  $\mathbf{y}$ ,  $\mathbf{k}(\mathbf{a})\mathbf{y} = \mathbf{a} \times \mathbf{y}$ , where  $\times$  denotes the vector cross product.

With this notation, we can write the time derivative of  $T_j$  in the form:

$$\dot{T}_j = \mathbf{R}(\mathbf{y}_j) T_j \quad (3)$$

where

$$\mathbf{y}_j = \mathbf{y}_0 + \sum_{i=1}^3 \mathbf{z}_i \dot{q}_i + \mathbf{z}_j \dot{q}_j$$

and,

$$\begin{aligned} \mathbf{v}_0 &= \begin{bmatrix} \mathbf{0} \\ \dot{\mathbf{p}}_0 \end{bmatrix} \\ \mathbf{p}_0 &= \begin{bmatrix} \mathbf{p}_0 \\ 1 \end{bmatrix} \end{aligned}$$

The vector  $\mathbf{p}_0$  is the position vector of the Floating Reference Frame and, in general, for any joint  $k$ ,

$$\mathbf{s}_k = \begin{cases} \begin{bmatrix} \mathbf{n}_k \\ \mathbf{r}_k \times \mathbf{n}_k \\ \mathbf{0} \\ \mathbf{n}_k \end{bmatrix} & \text{if joint } k \text{ is rotational} \\ \begin{bmatrix} \mathbf{r}_k \\ \mathbf{n}_k \end{bmatrix} & \text{if joint } k \text{ is translational} \end{cases} \quad (4)$$

where  $\mathbf{0}$  is the null vector and  $\mathbf{n}_k$  is a unit vector along the axis of rotation if the joint is rotational, or a unit vector in the direction of translation if the joint is translational. The vector  $\mathbf{r}_k$  is a position vector of an arbitrary point on the axis of rotation of the joint if rotational. The vectors  $\mathbf{n}_k$ ,  $\mathbf{r}_k$ , and  $\mathbf{p}_0$  are defined relative to the inertial coordinate frame.

With this notation we can write:

$$K_j = \frac{1}{2} T R \{ R(\mathbf{v}_j) T_j D_j T_j^T R(\mathbf{v}_j)^T \}$$

The matrix  $T_j D_j T_j^T$  is the pseudo inertia matrix of the  $j$ -th reaction wheel referred to the inertial coordinate frame. We now partition  $T_j D_j T_j^T$  into two parts:

$$T_j D_j T_j^T = N_j^* + N_j$$

where

$$\begin{aligned} N_j^* &= J_j^* \mathbf{n}_j \mathbf{n}_j^T + m_j \mathbf{r}_j \mathbf{r}_j^T \\ N_j &= \frac{1}{2} J_j (\mathbf{I} - \mathbf{e} \mathbf{e}^T - 2 \mathbf{n}_j \mathbf{n}_j^T) \end{aligned}$$

where  $J_j$  is the moment of inertia of the reaction wheel about its axis of rotation,  $J_j^*$  is the moment of inertia about an axis orthogonal to the rotational axis, and

$$\begin{aligned} \mathbf{n}_j &= \begin{bmatrix} \mathbf{n}_j \\ 0 \end{bmatrix} \\ \mathbf{r}_j &= \begin{bmatrix} \mathbf{r}_j \\ 1 \end{bmatrix} \end{aligned}$$

With this partitioning we note that

$$R(\mathbf{s}_j) N_j^* \equiv \mathbf{0}$$

and since,

$$R(\mathbf{v}_j) = R(\mathbf{v}_3) + R(\mathbf{s}_j) \dot{q}_j$$

we get:

$$K_j = \frac{1}{2} T R \{ R(\mathbf{v}_3) N_j^* R(\mathbf{v}_3)^T \} + \frac{1}{2} T R \{ R(\mathbf{v}_j) N_j R(\mathbf{v}_j)^T \}$$

From equation 1:

$$\begin{aligned} l_j &= \frac{\partial K}{\partial \dot{q}_j} \\ &= T R \{ R(\mathbf{s}_j) T_j D_j \dot{T}_j^T \} \\ &= J_j (\mathbf{n}_j^T \omega_3 + \dot{q}_j) \end{aligned}$$

where  $\omega_3$  is the angular velocity of the Base.

$$\omega_3 = \sum_{i=1}^3 \mathbf{n}_i \dot{q}_i$$

Direct expansion reveals that:

$$l_j^2 = J_j T R \{ R(\mathbf{v}_j) N_j R(\mathbf{v}_j)^T \}$$

Thus, we can write:

$$K_j = \frac{1}{2} T R \{ R(\mathbf{v}_3) N_j^* R(\mathbf{v}_3)^T \} + \frac{1}{2} J_j^{-1} l_j^2$$

Finally, we note that

$$\mathbf{E}_j = T_3^{-1} N_j^* (T_3^{-1})^T$$

is a constant matrix. We can therefore write:

$$K_j = \frac{1}{2} T R \{ \dot{T}_3 \mathbf{E}_j \dot{T}_3^T \} + \frac{1}{2} J_j^{-1} l_j^2$$

which is the desired result.

This allows us to rewrite the total kinetic energy in terms of the generalized momentum of the reaction wheels. We obtain,

$$K = \sum_{i=3}^{n+3} \frac{1}{2} T R \{ \dot{T}_i \mathbf{D}_i \dot{T}_i^T \} + \sum_{j=n+4}^{n+6} \frac{1}{2} J_j^{-1} l_j^2 \quad (5)$$

where

$$\mathbf{D}_i = \begin{cases} \mathbf{D}_i + \mathbf{E}_{n+4} + \mathbf{E}_{n+5} + \mathbf{E}_{n+6} & \text{if } i = 3 \\ \mathbf{D}_i & \text{if } i \neq 3 \end{cases}$$

It is of some interest to note that  $\mathbf{D}_3$  is the  $4 \times 4$  counterpart of the spatial articulated moment of inertia matrix, [12].

Note, for  $k \leq 3$ ,

$$\begin{aligned} \frac{\partial l_j}{\partial \dot{q}_k} &= T R \{ R(\mathbf{s}_j) T_j D_j T_j^T R(\mathbf{s}_k)^T \} \\ &= J_j (\mathbf{n}_j^T \mathbf{n}_k) \end{aligned}$$

and

$$\frac{\partial l_j}{\partial q_k} = J_j \mathbf{n}_j^T \dot{\mathbf{n}}_k = J_j \mathbf{n}_j (\omega_k \times \mathbf{n}_k)$$

So that,

$$\frac{d}{dt} \left( \frac{\partial l_j}{\partial \dot{q}_k} \right) - \frac{\partial l_j}{\partial q_k} = J_j \dot{\mathbf{n}}_j^T \mathbf{n}_k = J_j (\omega_3 \times \mathbf{n}_j)^T \mathbf{n}_k$$

For the manipulator joints,  $n+4 > k > 3$ ,

$$\frac{\partial l_j}{\partial \dot{q}_k} = \frac{\partial l_j}{\partial q_k} = \mathbf{0}$$

## 2.2 Elimination of Base Velocity

The form of the equation for the kinetic energy of the system given in equation 5 is defined in terms of the velocities relative to the inertial coordinate frame. Our objective in this section is to rewrite this equation in terms of velocities relative to Floating Reference Frame. That is, we will eliminate the term  $\dot{\mathbf{p}}_0$  found in equation 5. This is done by rewriting this equation in terms of the velocity of the system center of mass.

We begin by noting that:

$$\mathbf{T}_i = \mathbf{T}_0 \mathbf{A}_i$$

where  $A_i$  is the homogeneous transform of link  $i$  coordinates with respect to the Floating Reference Frame. Thus,

$$\begin{aligned}\dot{T}_i &= \dot{T}_0 A_i + T_0 \dot{A}_i \\ &= \dot{p}_0 e^T + \dot{A}_i\end{aligned}\quad (6)$$

where  $e = [0 \ 0 \ 0 \ 1]^T$ . Note that:

$$T_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 & p_0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The position of the center of mass of link  $j$  is given by:

$$p_j^c = T_j r_j^c$$

where,  $r_j^c = \text{constant}$ , is the location of the center of mass of link  $j$  with respect to link  $j$  coordinates. This is illustrated in figure 1. Note that  $r_3^c$  is the center of mass of the combination of the Base and the three reaction wheels. From equation 6 we get:

$$\dot{p}_j^c = \dot{T}_j r_j^c = \dot{p}_0 + \dot{A}_j r_j^c$$

By definition of the system center of mass, we have:

$$\begin{aligned}m_T \dot{p}^c &= \sum_{j=3}^{n+3} m_j \dot{p}_j^c = \sum_{j=3}^{n+3} m_j \dot{T}_j r_j^c \\ &= \sum_{j=3}^{n+3} m_j \dot{p}_0 + \sum_{j=3}^{n+3} m_j \dot{A}_j r_j^c \\ &= m_T \dot{p}_0 + \sum_{j=3}^{n+3} m_j \dot{A}_j r_j^c\end{aligned}$$

where  $m_j$  is the mass of link  $j$  and  $m_T$  is the total mass of the system. So the linear velocity of the Floating Frame is:

$$\dot{p}_0 = - \sum_{j=3}^{n+3} \frac{m_j}{m_T} \dot{A}_j r_j^c + \dot{p}^c$$

Substituting this into equation 6 gives:

$$\begin{aligned}\dot{T}_i &= \dot{p}^c e^T + \dot{A}_i - \sum_{j=3}^{n+3} \frac{m_j}{m_T} \dot{A}_j r_j^c e^T \\ &= \dot{p}^c e^T + \sum_{j=3}^{n+3} \dot{A}_j C_{ij}\end{aligned}$$

where

$$C_{ij} = \begin{cases} I - \frac{m_j}{m_T} r_j^c e^T & \text{if } i = j \\ -\frac{m_j}{m_T} r_j^c e^T & \text{if } i \neq j \end{cases}$$

Substituting this into equation 5 gives:

$$\begin{aligned}K &= \sum_{i=3}^{n+3} \frac{1}{2} TR\{\dot{T}_i D_i \dot{T}_i^T\} + \sum_{j=n+4}^{n+6} \frac{1}{2} J_j^{-1} \dot{l}_j^2 \\ &= \sum_{i=3}^{n+3} \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR\{\dot{A}_j C_{ij} D_i C_{ik}^T \dot{A}_k^T\} \\ &\quad + \sum_{i=3}^{n+3} \sum_{j=3}^{n+3} \frac{1}{2} TR\{\dot{A}_j C_{ij} D_i e(\dot{p}^c)^T\} \\ &\quad + \sum_{i=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR\{\dot{p}^c e^T D_i C_{ik}^T \dot{A}_k^T\}\end{aligned}$$

$$\begin{aligned}&+ \sum_{i=3}^{n+3} \frac{1}{2} TR\{\dot{p}^c e^T D_i e(\dot{p}^c)^T\} \\ &+ \sum_{j=n+4}^{n+6} \frac{1}{2} J_j^{-1} \dot{l}_j^2\end{aligned}$$

and since,

$$\sum_{i=3}^{n+3} C_{ij} D_i e(\dot{p}^c)^T = 0$$

We obtain,

$$K = \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR\{\dot{A}_j U_{jk} \dot{A}_k^T\} + \sum_{j=n+4}^{n+6} \frac{1}{2} J_j^{-1} \dot{l}_j^2 + g \quad (7)$$

where

$$\begin{aligned}U_{jk} &= \sum_{i=3}^{n+3} C_{ij} D_i C_{ik}^T \\ &= \begin{cases} D_j - \frac{m_j^2}{m_T} r_j^c (r_j^c)^T & \text{if } j = k \\ -\frac{m_j m_k}{m_T} r_j^c (r_k^c)^T & \text{if } j \neq k \end{cases} \\ g &= \sum_{i=3}^{n+3} \frac{1}{2} TR\{\dot{p}^c e^T D_i e(\dot{p}^c)^T\} \\ &= \frac{1}{2} m_T (\dot{p}^c)^T \dot{p}^c\end{aligned}$$

## 2.3 Lagrange's Equation

Lagrange's equation is used to obtain the dynamic equations of motion.

$$\tau_i = \frac{d}{dt} \frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i}$$

where  $\tau_i$  is the actuator torque if a rotational joint or actuator force if a translational joint. For the reaction wheels, this equation is particularly easy to evaluate since the position variables are cyclic. That is:

$$\frac{\partial K}{\partial q_i} = 0$$

So that,

$$\tau_i = \frac{dl_i}{dt}$$

where  $\tau_i$  is the actuator torque of the  $i$ -th reaction wheel.

For the remaining variables, we note that:

$$\frac{\partial \dot{A}_j}{\partial \dot{q}_i} = \frac{\partial A_j}{\partial q_i}$$

and

$$\frac{d}{dt} \left( \frac{\partial \dot{A}_j}{\partial \dot{q}_i} \right) = \frac{\partial \dot{A}_j}{\partial q_i}$$

Hence,

$$\begin{aligned}\frac{\partial K}{\partial \dot{q}_i} &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR\left\{\frac{\partial \dot{A}_j}{\partial \dot{q}_i} U_{jk} \dot{A}_k^T\right\} \\ &\quad + \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR\left\{\dot{A}_j U_{jk} \left(\frac{\partial \dot{A}_k}{\partial \dot{q}_i}\right)^T\right\} \\ &\quad + \sum_{j=n+4}^{n+6} \left(\frac{1}{J_j} \frac{\partial \dot{l}_j}{\partial \dot{q}_i}\right)\end{aligned}$$

$$= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial \dot{A}_j}{\partial \dot{q}_i} U_{jk} \dot{A}_k^T \right\} + \sum_{j=n+4}^{n+3} \left( \frac{1}{J_j} l_j \frac{\partial l_j}{\partial \dot{q}_i} \right)$$

since  $U_{kj}^T = U_{jk}$ .

Similarly:

$$\begin{aligned} \frac{\partial K}{\partial q_i} &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR \left\{ \frac{\partial \dot{A}_j}{\partial q_i} U_{jk} \dot{A}_k^T \right\} \\ &+ \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR \left\{ \dot{A}_j U_{jk} \left( \frac{\partial \dot{A}_k}{\partial q_i} \right)^T \right\} \\ &+ \sum_{j=n+4}^{n+3} \left( \frac{1}{J_j} l_j \frac{\partial l_j}{\partial q_i} \right) \\ &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial \dot{A}_j}{\partial q_i} U_{jk} \dot{A}_k^T \right\} \\ &+ \sum_{j=n+4}^{n+3} \left( \frac{1}{J_j} l_j \frac{\partial l_j}{\partial q_i} \right) \end{aligned}$$

since  $U_{kj}^T = U_{jk}$ .

Therefore, the equations of motion are:

$$\tau_i = \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} U_{jk} \ddot{A}_k^T \right\} + u_i$$

where

$$u_i = \begin{cases} n_i^T u & \text{if } i \leq 3 \\ 0 & \text{if } 3 < i \leq n+3 \end{cases} \quad (8)$$

and

$$u = \sum_{j=n+4}^{n+6} \frac{d}{dt} (n_j l_j) = \sum_{j=n+4}^{n+6} (n_j \tau_j + \omega_3 \times n_j l_j) \quad (9)$$

Since there is no external torque applied to the Base, the  $\tau_i$  are zero for  $i < 3$ . Therefore, to simplify the development of the controller we make the following definitions. For  $0 < i < n+4$  let,

$$\rho_i = \tau_i - u_i \quad (10)$$

The equations of motion then become, for  $0 < i < n+4$ :

$$\rho_i = \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} U_{jk} \ddot{A}_k^T \right\} \quad (11)$$

For the remainder of this paper, we will consider  $\rho_i$  to be the inputs to the system. If the  $\rho_i$  are given, the actuator inputs  $\tau_j$  can be obtained from equations 8 through 10.

### 3 Adaptive Controller

In this section we present the control law and adaptation law so that the system tracks onto the desired joint trajectory. Global asymptotic stability is proven and a recursive formulation of the controller is provided for computational efficiency.

#### 3.1 Method of Control

The controller is a modified version of an inverse dynamic controller with adaptation. Let  $q$  be an  $(n+3) \times 1$  vector of the joint positions which includes the three orientation angles of the Base and the  $n$  joint angles of the manipulator. We start by defining the variable,  $\dot{q}$  in terms of the position and velocity errors,  $q_e = q - q_d$ . Let:

$$\dot{q} = \dot{q}_e + \lambda q_e \quad (12)$$

where  $\lambda$  is a positive definite diagonal matrix with positive diagonal components  $\lambda_i$  and  $q_d$  is the desired value of  $q$ . Thinking of  $\dot{q}$  as an input, this defines an exponentially stable and strictly proper transfer function between  $\dot{q}$  and  $q_e$ . The method of control is to select the control law and adaptation law such that  $\dot{q}$  is an  $L^2$  function. It can be shown that, [13]:

$$\text{if } \dot{q} \in L^2 \text{ then } \begin{cases} q_e \in L^2 \cap L^\infty \\ \dot{q}_e \in L^\infty \\ q_e \text{ is continuous} \\ q_e(t) \rightarrow 0 \text{ as } t \rightarrow \infty \end{cases}$$

Thus, we have proven that the position and velocity tracking errors have converged to zero if we can show that  $\dot{q}$  is an  $L^2$  function.

A judicious choice of the norm of  $\dot{q}$  is a critical part of the method. The following norm  $\tilde{K}$  is an appropriate choice:

$$\tilde{K} = \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR \left\{ \dot{A}_j U_{jk} \dot{A}_k^T \right\} > 0 \quad \forall \dot{q} \neq 0$$

where

$$\dot{A}_k = \sum_{i=1}^k \frac{\partial A_k}{\partial q_i} \dot{q}_i$$

The time derivative of  $\tilde{K}$  is:

$$\begin{aligned} \frac{d\tilde{K}}{dt} &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR \left\{ \ddot{A}_j U_{jk} \dot{A}_k^T \right\} \\ &+ \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \frac{1}{2} TR \left\{ \dot{A}_j U_{jk} \ddot{A}_k^T \right\} \\ &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \dot{A}_j U_{jk} \ddot{A}_k^T \right\} \end{aligned}$$

#### 3.2 The Controller

From equation 12 we get:

$$\dot{q} = \dot{q}_d + \lambda q_e \quad (13)$$

where  $\dot{q} = \dot{q} - \dot{q}$ . From this we get:

$$\ddot{q} = \ddot{q}_d + \lambda \dot{q}_e \quad (14)$$

Defining:

$$\dot{A}_k = \sum_{i=1}^k \frac{\partial A_k}{\partial q_i} \dot{q}_i$$

Then, the control law is:

$$\rho_i = \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} \dot{U}_{jk} (\ddot{A}_k - \gamma \dot{A}_k)^T \right\} \quad (15)$$

and the adaptation law is:

$$\dot{U}_{jk} = -\alpha_{jk} \dot{A}_j^T (\ddot{A}_k - \gamma \dot{A}_k) \quad (16)$$

where  $\gamma$  and  $\alpha_{jk}$  are positive constants, and  $\dot{U}_{jk}$  is current estimate of  $U_{jk}$ . Note that only  $\ddot{q}$  and  $\dot{q}$  are required for control and not  $\ddot{q}$ . Also, the acceleration,  $\ddot{q}$ , is not required.

### 3.3 Stability Proof

We define the nonnegative function  $V(t)$ :

$$V(t) = \tilde{K} + 1/2 \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \alpha_{jk}^{-1} TR \{ \tilde{U}_{jk} \tilde{U}_{jk}^T \} \quad (17)$$

where the  $\alpha_{jk}$  are positive constants, and  $\tilde{U}_{jk} = U_{jk} - \hat{U}_{jk}$  is the error in the estimate of  $U_{jk}$ .

To prove stability we first show that  $\dot{V}(t) \leq 0$ . We start by substituting equation 15 into the equations of motion, equation 11. This gives:

$$\begin{aligned} 0 &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} (U_{jk} \ddot{A}_k^T - \hat{U}_{jk} \ddot{A}_k^T) \right\} \\ &+ \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} \dot{U}_{jk} \dot{A}_k^T \right\} \\ &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} (U_{jk} (\ddot{A}_k^T - \dot{A}_k^T) + (\dot{U}_{jk} - \hat{U}_{jk}) \ddot{A}_k^T) \right\} \\ &+ \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} \dot{U}_{jk} \dot{A}_k^T \right\} \\ &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} (U_{jk} \ddot{A}_k^T + \hat{U}_{jk} \ddot{A}_k^T) \right\} \\ &+ \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} \dot{U}_{jk} \dot{A}_k^T \right\} \end{aligned}$$

Multiply by  $\dot{q}_i$  and summing over all  $i$  gives:

$$\begin{aligned} 0 &= \sum_{i=1}^{n+3} \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} (U_{jk} \ddot{A}_k^T + \hat{U}_{jk} \ddot{A}_k^T) \dot{q}_i \right\} \\ &+ \gamma \sum_{i=1}^{n+3} \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \left\{ \frac{\partial A_j}{\partial q_i} \dot{U}_{jk} \dot{A}_k^T \dot{q}_i \right\} \\ &= \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ (\dot{A}_j U_{jk} \ddot{A}_k^T + \dot{A}_j \hat{U}_{jk} \ddot{A}_k^T) \} \\ &+ \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \dot{A}_j \dot{U}_{jk} \dot{A}_k^T \} \\ &= \frac{d\tilde{K}}{dt} + \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \dot{A}_j \tilde{U}_{jk} \ddot{A}_k^T \} \\ &+ \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \dot{A}_j \hat{U}_{jk} \dot{A}_k^T \} \end{aligned}$$

Thus,

$$\begin{aligned} \frac{d\tilde{K}}{dt} &= - \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \tilde{U}_{jk} \ddot{A}_k^T \dot{A}_j \} \\ &- \gamma \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \hat{U}_{jk} \dot{A}_k^T \dot{A}_j \} \end{aligned}$$

Where we have used the trace identity  $TR\{ABC\} = TR\{BCA\}$  for any square matrices  $A, B$ , and  $C$ . Adding and subtracting  $2\gamma\tilde{K}$  gives:

$$\frac{d\tilde{K}}{dt} = - \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \tilde{U}_{jk} (\ddot{A}_k - \gamma \dot{A}_k)^T \dot{A}_j \} - 2\gamma\tilde{K} \quad (18)$$

Thus, if there are no errors in the parameter estimates,  $\tilde{U}_{jk} = 0$ , then  $\tilde{K}$  satisfies the linear equation,  $d\tilde{K}/dt + 2\gamma\tilde{K} = 0$  and, hence,  $\tilde{K}(t) = e^{-2\gamma t} \tilde{K}(0)$  and the stability result immediately follows. However, for the case in point, the parameter values are not initially known and we must proceed further.

From equation 17 we get:

$$\dot{V}(t) = \frac{d\tilde{K}}{dt} - \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \alpha_{jk}^{-1} TR \{ \tilde{U}_{jk} \dot{U}_{jk}^T \}$$

Substituting in equation 18 gives:

$$\begin{aligned} \dot{V}(t) &= - \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} TR \{ \tilde{U}_{jk} (\ddot{A}_k - \gamma \dot{A}_k)^T \dot{A}_j \} - 2\gamma\tilde{K} \\ &- \sum_{j=3}^{n+3} \sum_{k=3}^{n+3} \alpha_{jk}^{-1} TR \{ \tilde{U}_{jk} \dot{U}_{jk}^T \} \end{aligned}$$

Substituting in the adaptation equation 16 gives:

$$\dot{V}(t) = -2\gamma\tilde{K} \leq 0$$

Hence,  $0 \leq V(t) \leq V(0) < \infty$ , or  $0 \leq V(0) - V(t) < \infty$ .

From the equivalence of finite dimensional vector space norms, there exist a positive constant  $\beta$  such that:

$$\dot{q}^T \dot{q} \leq \beta \tilde{K}$$

Therefore,

$$\begin{aligned} \int_0^\infty \dot{q}^T \dot{q} dt &\leq \int_0^\infty \beta \tilde{K} dt \\ &= \frac{\beta}{2\gamma} \int_0^\infty -\dot{V}(t) dt \\ &= \frac{\beta}{2\gamma} [V(0) - V(\infty)] \\ &< \infty \end{aligned}$$

Therefore,  $\dot{q}$  is an  $L^2$  function and, hence,  $q_e$  converges to zero, which is the desired result.

### 3.4 Recursive Formulation of Controller

The computational efficiency of the control law is greatly improved by writing the equations in a recursive form. First we define:

$$R(s_k) = T_0^{-1} R(\underline{z}_k) T_0$$

This matrix  $R(s_k)$  is simply  $R(\underline{s}_k)$  referred to the Floating Reference Frame. The vector  $s_k$  has the same interpretation as  $\underline{s}_k$  defined in equation 4 except that all the vectors are defined with respect to the floating reference frame. Let:

$$R(v_j) = T_0^{-1}(R(\underline{v}_k) - R(\underline{v}_0))T_0 = \sum_{i=1}^j R(s_i)\dot{q}_i$$

Then for  $\dot{A}_k$  we have:

$$\begin{aligned}\dot{A}_k &= R(v_k)A_k \\ R(v_k) &= R(v_{k-1}) + R(s_k)\dot{q}_k \\ R(v_0) &= 0\end{aligned}$$

For  $\dot{\tilde{A}}_k$ :

$$\begin{aligned}\dot{\tilde{A}}_k &= R(\tilde{v}_k)A_k \\ R(\tilde{v}_k) &= R(\tilde{v}_{k-1}) + R(s_k)\dot{\tilde{q}}_k \\ R(\tilde{v}_0) &= 0\end{aligned}$$

and for  $\dot{\hat{A}}_k$ :

$$\begin{aligned}\dot{\hat{A}}_k &= R(\hat{v}_k)A_k \\ R(\hat{v}_k) &= R(\hat{v}_{k-1}) + R(s_k)\dot{\hat{q}}_k \\ R(\hat{v}_0) &= 0\end{aligned}$$

For,  $\ddot{A}_k$ :

$$\begin{aligned}\ddot{A}_k &= (\dot{R}(\tilde{v}_k) + R(\tilde{v}_k)R(v_k))A_k \\ \dot{R}(\tilde{v}_k) &= \dot{R}(\tilde{v}_{k-1}) + R(s_k)\ddot{\tilde{q}}_k + \dot{R}(s_k)\dot{\tilde{q}}_k \\ &= \dot{R}(\tilde{v}_{k-1}) + R(s_k)\ddot{\tilde{q}}_k \\ &\quad + (R(v_k)R(s_k) - R(s_k)R(v_k))\dot{\tilde{q}}_k \\ \dot{R}(\tilde{v}_0) &= 0\end{aligned}$$

These, equations are computed recursively from the Floating Reference Frame to the end-effector, link  $n+3$ .

Next we define:

$$\begin{aligned}F_{jk} &= A_j \dot{U}_{jk}(\ddot{A}_k - \gamma \dot{A}_k)^T \\ F_j &= \sum_{k=3}^{n+3} F_{jk}\end{aligned}$$

and

$$\begin{aligned}f_i &= \sum_{j=i}^{n+3} F_j = F_i + f_{i+1} \\ f_{n+4} &= 0\end{aligned}$$

This equation is computed recursively from the end-effector, link  $i = n+3$  to the Floating Reference Frame.

Finally, the input is computed:

$$\rho_i = TR\{R(s_i)f_i\} \quad (19)$$

Note that:

$$\frac{\partial A_j}{\partial q_i} = \begin{cases} R(s_i)A_j & \text{if } i \leq j \\ 0 & \text{if } i > j \end{cases}$$

## 4 Conclusion

An efficient algorithm for the adaptive control of a space based robot has been presented. The method makes no assumptions on the initial estimates of the inertial parameters or the initial momentum of the system. Only the position and velocities of the manipulator joints and the Base orientation angles are required by the controller.

The first part of the paper develops the dynamic equations of motion for the system. Key to the method is the use of reaction wheels to control the orientation of the Base and the elimination of the Base linear motion from the equations of motion. It was shown that the effect of the reaction wheels on the dynamics of the system can be divided into two components. The first was the component related to the generalized momentum of the reaction wheels. The remaining component can be effectively included in the dynamics by modifying the inertial properties of the Base. The linear motion of the Base was removed from the equations of motion by using the law of conservation of linear momentum. With these two transformations, the resulting form of the kinetic energy was easily utilized in Lagrange's equation to obtain the dynamic equations of motion.

The algorithm used in the implementation of the adaptive controller is a modification of that presented for terrestrial based manipulators [14]. The primary differences are due to the use of homogeneous transforms in the equations of motion. For example, inner products of vectors become inner products of matrices. Another difference is the choice of the vector norm used in the stability proof. For terrestrial based manipulators this is directly related to the total kinetic energy of the manipulator. For the space based manipulator, this was related to the total kinetic energy minus the component due to the generalized momentum of the reaction wheels and the translational kinetic energy component.

A recursive form of the control algorithm was presented for computational efficiency. The computational load is still fairly high and increases quadratically in the number of links in the system. Due to the coupling of the dynamics, there does not seem to be any way to avoid the quadratic complexity problem. Since it is known that  $U_{jk} = U_{kj}^T$ , some computational improvements could be made with a slight modification of the adaptation law such that only  $U_{jk}$  is estimated instead the current method of estimating both  $U_{jk}$  and  $U_{kj}$ . However, this does not appear to produce very significant improvements. A more promising approach might be to avoid all of the coordinate transformations by referring the velocities, accelerations and forces to their own link coordinates. For example, one would compute  $A_k^{-1}\dot{A}_k$  instead of  $\dot{A}_k$  and  $A_j^{-1}F_{jk}(A_k^{-1})^T$  instead of  $F_{jk}$ .

A significant extension of these results would be the solution for manipulators containing closed kinematic loops, since these are the most common types of manipulators encountered in practice. This would also lead to methods for dual arm coordinated motion control and compliant motion control. Another extension would be an adaptive Cartesian coordinate controller. This allow two manipulators mounted on different bases to work in a coordinated manner.

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